Noise-Resilient Quantum Simulation with Quantum Error Detection Code

Dawei Zhong, Todd A. Brun

University of Southern California, Los Angeles, CA

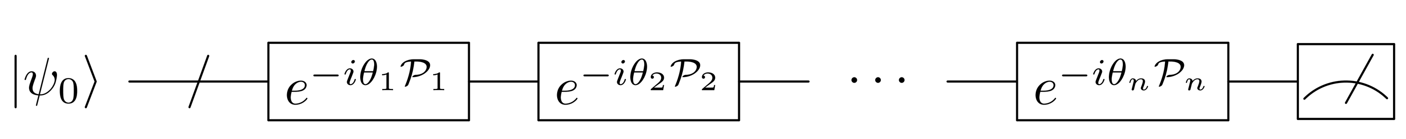
Assumptions and Notations

We create circuits with extra ancillas to protect $\exp(-i\theta\mathcal{P})$ under $[[n, n-2, 2]]$ code

- **Assumptions**. We consider gate based Pauli noise but noiseless encoding, decoding and mid-circuit syndrome measurements. We perform analysis on fully connected hardware and assume that the delay due to mid-circuit syndrome measurements will not bring any error during idle time.
- **Notation**. $\overline{\mathcal{P}}$ for the logical operator and $\mathcal{P}$ for physical operator corresponding to $\overline{\mathcal{P}}$. I, X, Y, Z are single qubit Pauli operators.

Circuits with Exponential Operations

- For fermionic system simulation, convert the Hamiltonian into Pauli form using Jordan-Wigner, Bravyi-Kitaev or parity mapping to achieve $H \xrightarrow{\text{mapping}} \tilde{H}$, where $\tilde{H} = \sum_j \alpha_j \mathcal{P}_j$.
- Use product formula approach to obtain $e^{-i\theta\tilde{H}} \simeq \prod_j e^{-i\theta_j\mathcal{P}_j}$ where $\theta_j = \alpha_j\delta_j$.
- Some parameterized circuits (e.g., QAOA and UCC ansatz) also have similar structure.
- Create circuits in the following form:



Quantum Error Detection Code and Weakly Fault Tolerance

The stabilizer generators are $X^{\otimes n}$ and $Z^{\otimes n}$. Logical Pauli operators are given by $\overline{X}_i = X_iX_{n-1}$, $\overline{Z}_i = Z_iZ_n$ and $\overline{Y}_i = i\overline{X}_i\overline{Z}_i$. The logical rotation gates are

$$\overline{R_{X_i}}(\theta) = \exp\left(-i\frac{\theta}{2}X_iX_{n-1}\right), \overline{R_{Z_i}}(\theta) = \exp\left(-i\frac{\theta}{2}Z_iZ_{n-1}\right) \quad (1)$$

Below are examples of logical rotations under the $[[4, 2, 2]]$ code.

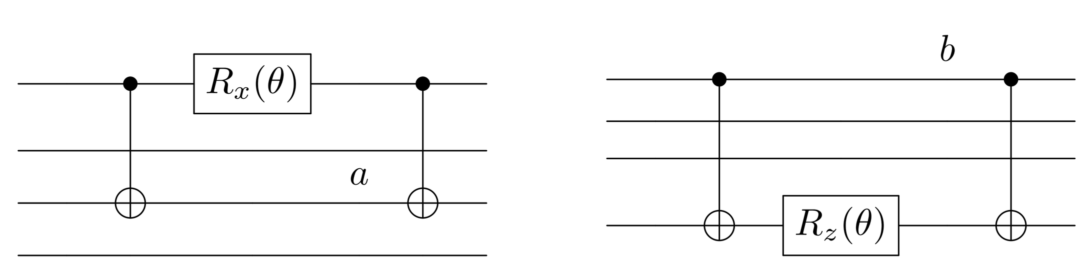


Figure 1. Circuit of $\overline{R_{X_1}}(\theta) = \exp(-i\theta X_1X_3/2)$ (left) and $\overline{R_{Z_1}}(\theta) = \exp(-i\theta Z_1Z_4/2)$ (right).

They are not fault-tolerant, since a Z_3 error occurring at location a becomes a Z_2Z_4 error, and a X_1 error at location b becomes a X_1X_4 error, both undetectable.

Weakly Fault Tolerant Rotation

Previous research (not yet published) from Todd Brun and Christopher Gerhard purposes a weakly fault-tolerant construction shown in Fig. 2 that helps detect most single- and two-qubits Pauli noise, except for X error at location d and Z error at location f, both of which relate to imprecise rotation angle $\theta + \Delta\theta$.

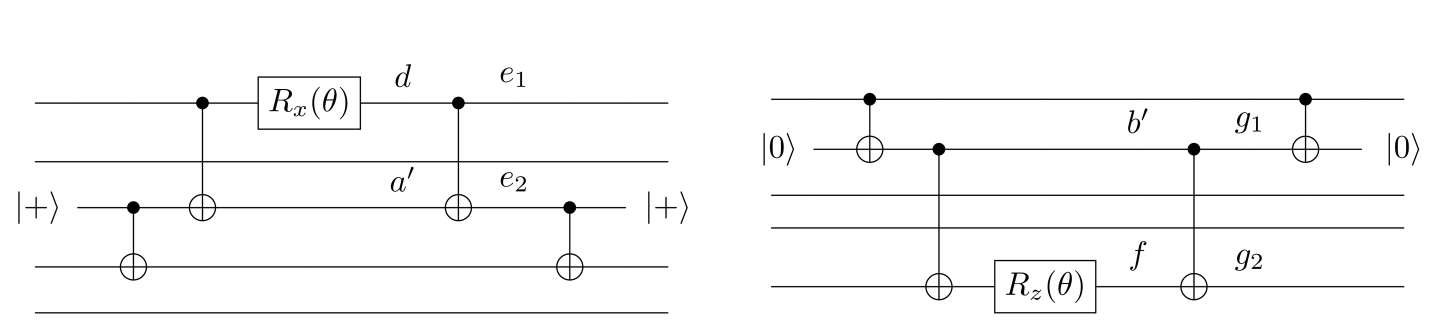


Figure 2. Circuit of weakly fault tolerant $\overline{R_{X_1}}(\theta)$ (left) and $\overline{R_{Z_1}}(\theta)$ (right)

Approach to Construct Noise-Resilient Operation

Inspired by adding ancilla to create weakly fault-tolerance, we purpose four steps to create noise-resilient exponential map $\exp(-i\theta\overline{\mathcal{P}})$ with $[[n, n-2, 2]]$ code. Noise-resilience is worse than weakly fault-tolerance but better than doing nothing.

Step 1: Construct Physical Operator

A physical operator of $\exp\left(-i\theta\overline{\mathcal{P}}\right)$ can be done by $\exp\left(-i\theta\overline{\mathcal{P}}\right) \rightarrow \exp\left(-i\theta\mathcal{P}\right)$, where $\mathcal{P}$ is physical operator of $\overline{\mathcal{P}}$ under $[[4, 2, 2]]$ code.

Step 2: Add ancilla to Physical Operator

Adding ancilla that could **decrease the number of remaining logical error** at the end of circuit but do not change expected operation.

Step 3: Create Equivalent Circuits

Create a group of circuits that equivalently implement $\exp\left(-i\theta\overline{\mathcal{P}}\right)$ with one extra ancilla. **Different circuits have different noise-resilient performance**.

Step 4: Search for Best Noise-Resilient Circuit

Search for circuits with the least remaining logical error (best noise-resilient performance) after syndrome measurement.

Examples

Logical Rotation-Y

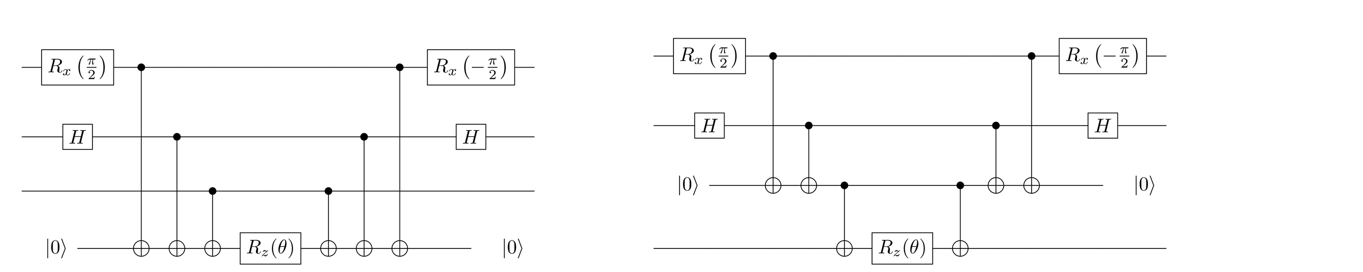


Figure 3. Two noise-resilient circuits that implement $\overline{R_{Y_i}}(\theta) = \exp\left(-i\theta Y_iX_{n-1}Z_n/2\right)$ in $[[n, n-2, 2]]$.

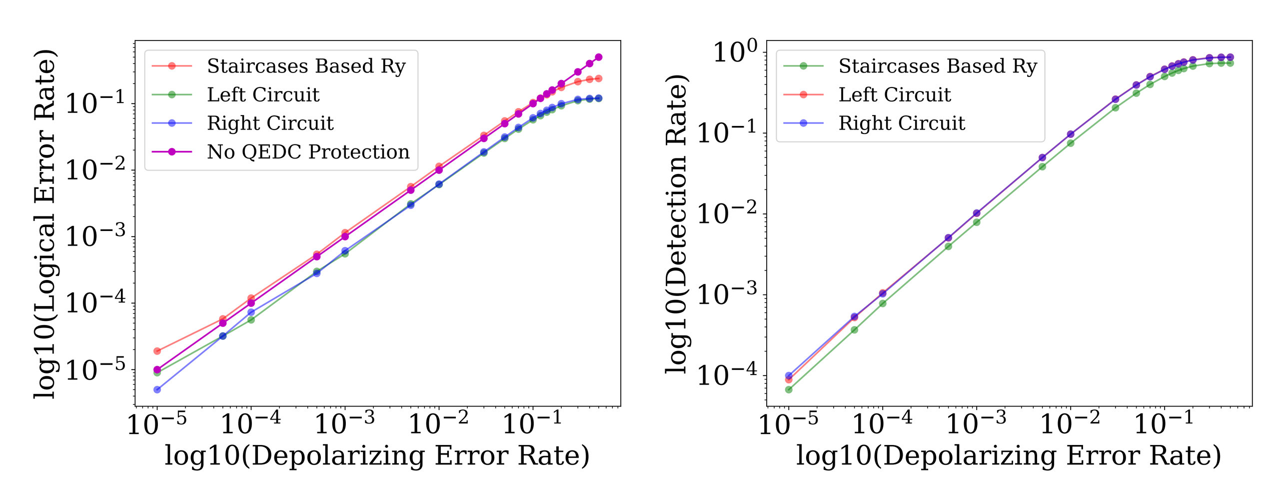


Figure 4. Logical error rate (left) and post-selection rate (right) of two noise-resilient circuits under 1,000,000 trials simulation with depolarizing gate noise.

Logical Exponential with Weight $n > 3$ Pauli

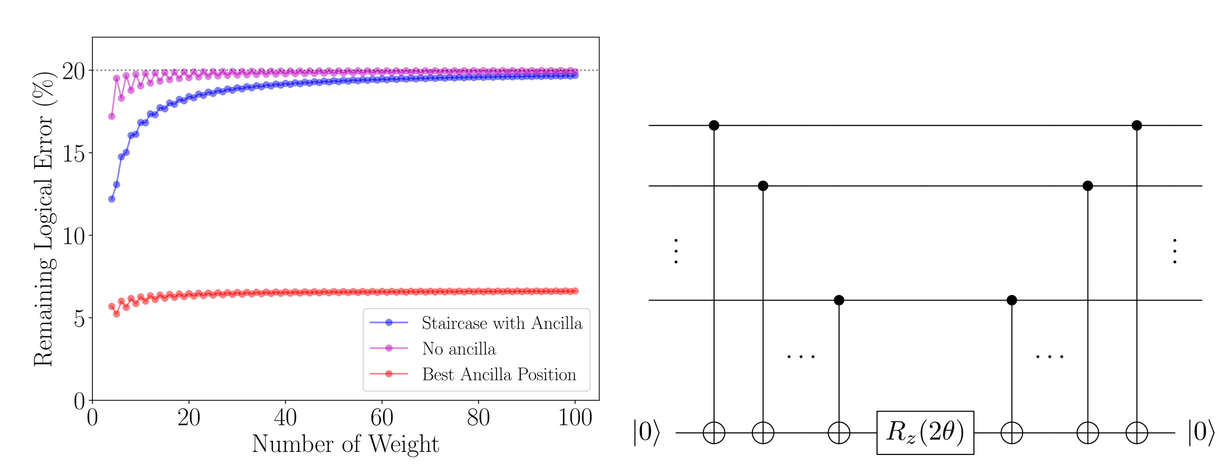
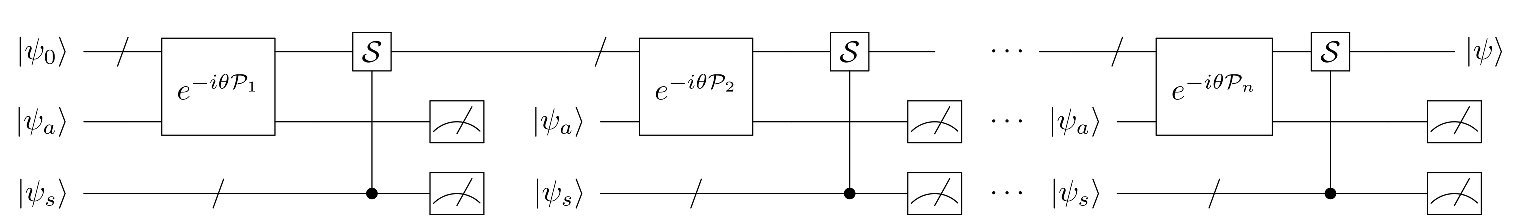


Figure 5. Percentage of remaining logical error versus weights in $\exp(-i\theta Z^{\otimes n})$ using circuit on the right.

With Mid-Circuit Syndrome Measurement

Protocol

Use noise-resilient circuit for $\exp(-i\theta\mathcal{P})$ and measure syndromes after each layer.



Performance Under Error Rate p with k Logical Qubits

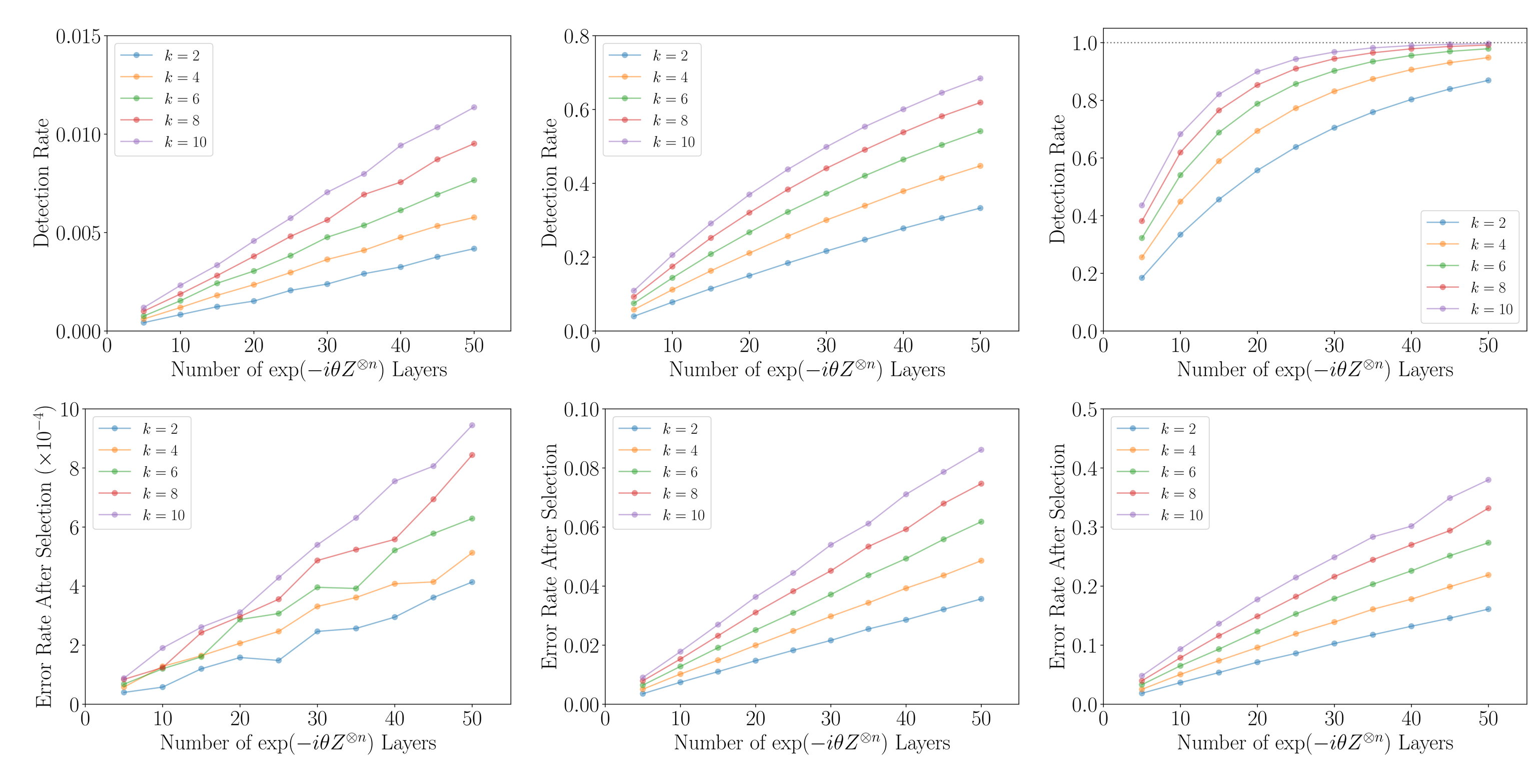


Figure 6. Performance with different number of $\exp(-i\theta Z^{\otimes n})$ layer ($n = k + 2$) with $p = 10^{-5}$ (left), 10^{-3} (middle) and 5×10^{-3} (right).

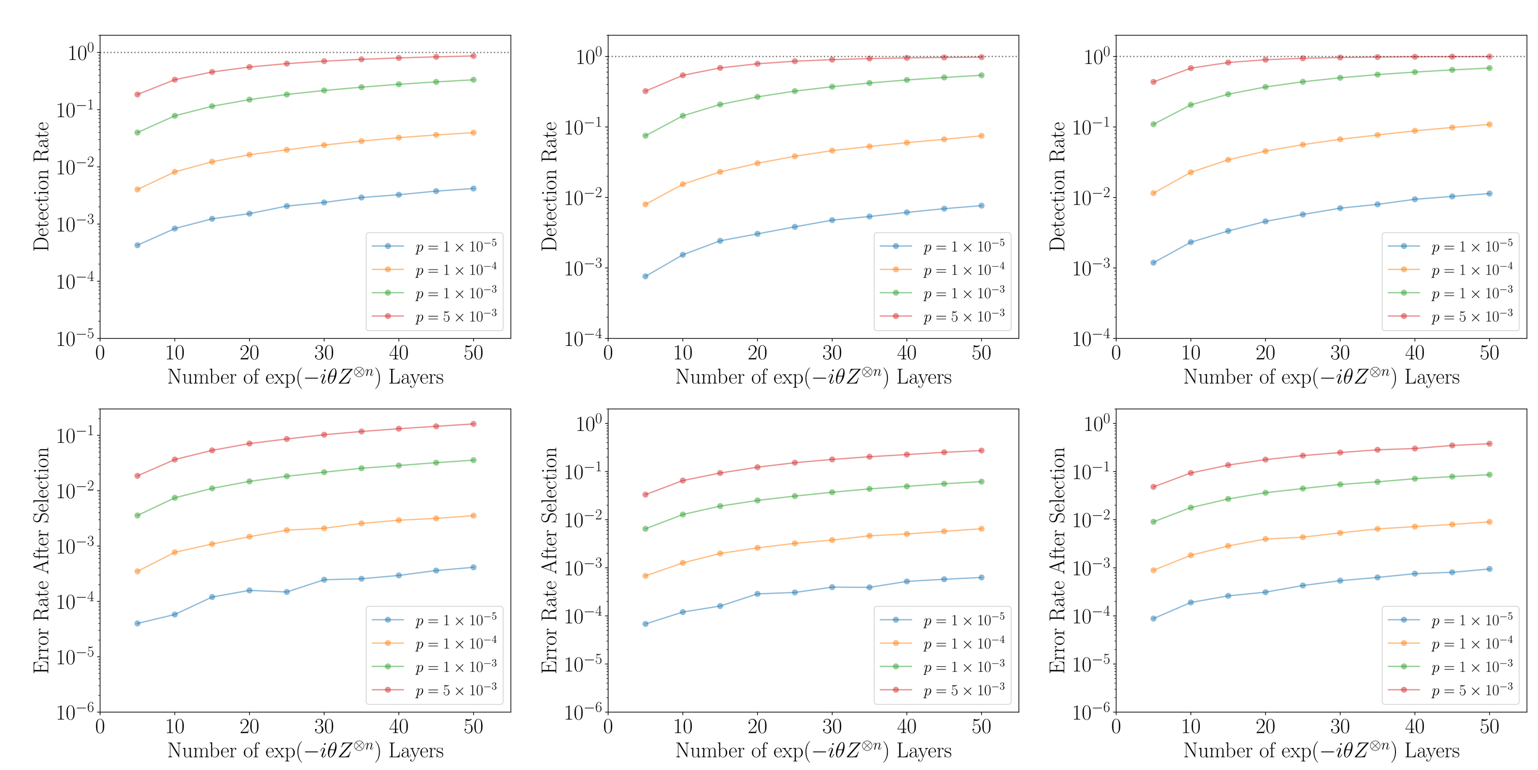


Figure 7. Performance with different number of $\exp(-i\theta Z^{\otimes n})$ layer ($n = k + 2$) with logical qubits number $k = 2$ (left), 6 (middle) and 10 (right).

Comments

1. Advantages of our approach:
   - Easy to implement in near-term devices.
   - Optimize near term resources by using fewer qubits and $\mathcal{O}(2n)$ CNOTs.
   - Reduce overhead by discarding noisy shots and no need for correction compare with QEC.
   - Can be used on near-term small-sacle fermionic system simulation.
   - It can be easily extended for any $[[n, k, d]]$ stabilizer code.
2. Disadvantages of using our approach
   - Required device with full connectivity to achieve optimal performance (e.g, ion trap system)
   - Required device with perfect qubit reset to achieve best mid-circuit measurement.

Contact Information: daweiz@usc.edu for Dawei Zhong and tbrun@usc.edu for Todd Brun.