# Noise-Resilient Quantum Simulation with Quantum Error Detection Code

Dawei Zhong, Todd A. Brun

University of Southern California, Los Angeles, CA

## Background

In this work, we create circuits with extra ancillas to protect the logical exponential operator by a quantum error detecting code.

- **Assumptions**. We consider Pauli noise after gates and hardware with full connectivity and mid-circuit measurements.
- **Notation**. We use $\overline{\mathcal{P}}$ for the logical operator and $\mathcal{P}$ to represent the physical operator corresponding to $\overline{\mathcal{P}}$. $I, X, Y, Z$ are single qubit Pauli operators.

### Product Formula Simulation

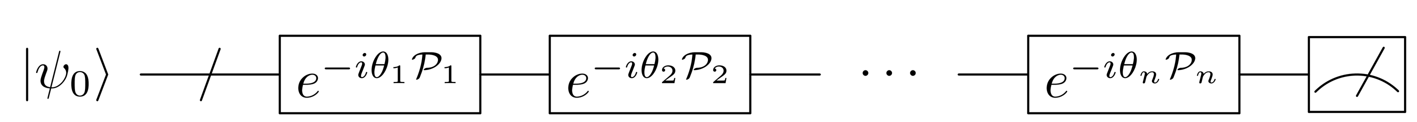
- For fermionic system simulation, convert the Hamiltonian into Pauli form using Jordan-Wigner, Bravyi-Kitaev or parity mapping

$$H \xrightarrow{\text{mapping}} \tilde{H}, \text{ where } \tilde{H} = \sum_j \alpha_j \mathcal{P}_j \tag{1}$$

- Use product formula approach (Trotter-Suzuki decomposition) to decompose

$$e^{-i\theta\tilde{H}} \simeq \prod_j e^{-i\theta_j \mathcal{P}_j}, \text{ where } \theta_j = \alpha_j \theta \tag{2}$$

- Create circuits in the following form:




### Quantum Error Detection Code

The stabilizer generators are $X^{\otimes n}$ and $Z^{\otimes n}$. Logical Pauli operators are given by $\overline{X}_i = X_i X_{n-1}$, $\overline{Z}_i = Z_i Z_n$ and $\overline{Y}_i = i\overline{X}_i\overline{Z}_i$. The logical rotation gates are

$$\overline{R_{X_i}}(\theta) = \exp\left(-i\frac{\theta}{2}X_i X_{n-1}\right), \overline{R_{Z_i}}(\theta) = \exp\left(-i\frac{\theta}{2}Z_i Z_{n-1}\right) \tag{3}$$

Below are examples of logical rotations under the $[[4,2,2]]$ code.

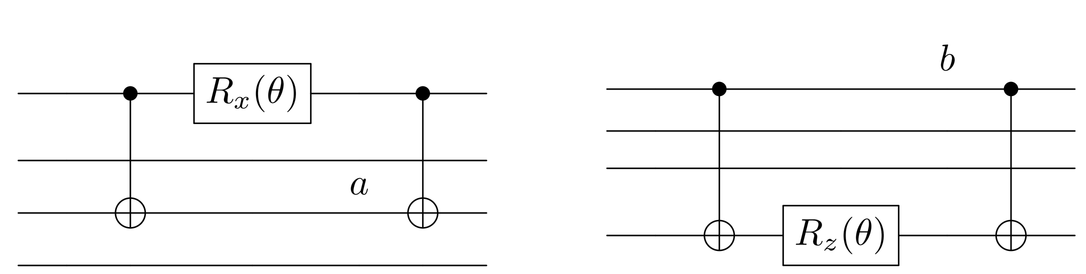



Figure 1. Circuit of $\overline{R_{X_1}}(\theta) = \exp(-i\theta X_1 X_3/2)$ (left) and $\overline{R_{Z_1}}(\theta) = \exp(-i\theta Z_1 Z_4/2)$ (right).

They are not fault-tolerant, since a $Z_3$ error occurring at location $a$ becomes a $Z_2 Z_4$ error, and a $X_1$ error at location $b$ becomes a $X_1 X_4$ error, both undetectable.

### Weakly Fault Tolerant Rotation

Previous research (not yet published) from Todd Brun and Christopher Gerhard purposes a weakly fault-tolerant construction shown in Fig. 2 that helps detect most single- and two-qubit Pauli noise, except for $X$ error at location $d$ and $Z$ error at location $f$.

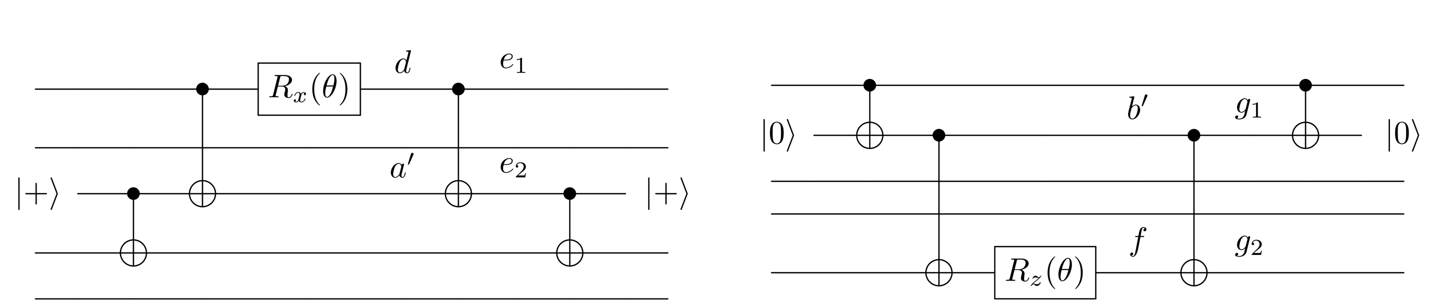



Figure 2. Circuit of weakly fault tolerant $\overline{R_{X_1}}(\theta)$ (left) and $\overline{R_{Z_1}}(\theta)$ (right)

## Approach

Four steps to protect each exponential map $\exp(-i\theta\overline{\mathcal{P}})$ with $[[n, n-2, 2]]$ code.

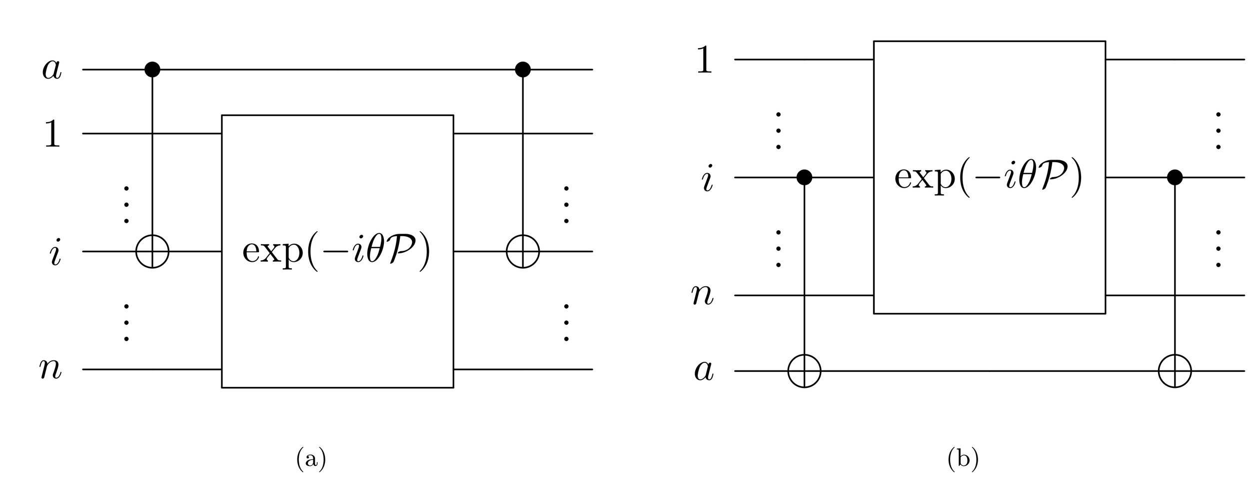
### Step 1: Construct Physical Operator

Construct physical operator by $\exp\left(-i\theta\overline{\mathcal{P}}\right) \rightarrow \exp\left(-i\theta\mathcal{P}\right)$, which fulfill two requirements.

- The given circuit should perform logical operation $\overline{U}$ on logical state $|\overline{\psi}\rangle$
- The given circuit should always convert any valid logical state (i.e., a state that falls into the codespace) into another valid logical state in same subspace.

### Step 2: Create Equivalent Circuits

Create a group of circuits that equivalently implement $\exp\left(-i\theta\mathcal{P}\right)$ with one (or more) ancilla. Use the following construction to create equivalent circuits:




Here the $i$–th qubit **can be any qubits** between qubit 1 and qubit $n$. The new map $U$ is still an exponential map $\exp(-i\theta\mathcal{P}')$, and we can set every possible qubit as ancilla.

### Step 3: Search for Noise-Resilient circit

The most-noise resilient circit is the one with the least logical error ratio (`num_logical_error/num_total_err`) under all possible depolarizing Pauli noise after each gate. Use numerical simulation to find the noise-resilient circit.

### Step 4: Perform Mid-circuit syndrome Measurement

Use noise-resilient operator construction for $\exp(-i\theta\mathcal{P})$. After measuring error syndrome after each $\exp(-i\theta\mathcal{P})$, we discard the noisy shots and use the remaining shots for further analysis.

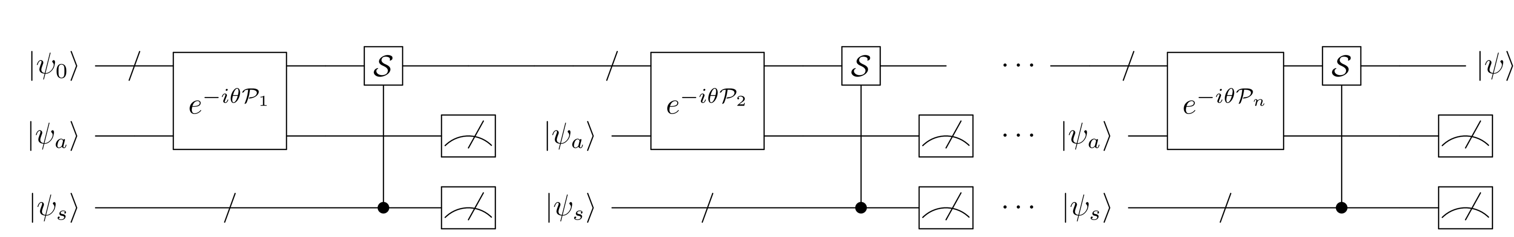



Figure 3. Mid-Circuit Measurement Protocol.

## Comments

1. Advantages of our approach:
   - Easy to implement in near-term devices.
   - Optimize near term resources by using fewer qubits and $\mathcal{O}(2n)$ CNOTs.
   - Reduce overhead by discarding noisy shots and no need for correction compare with QEC.
   - Can be used on near-term small-sacle fermionic system simulation.
   - It can be easily extended for any $[[n, k, d]]$ stabilizer code.
2. Disadvantages of using our approach
   - Required device with full connectivity to achieve optimal performance (e.g, ion trap system)
   - Required device with perfect qubit reset to achieve best mid-circuit measurement.

## Examples and Performance

### Logical Rotation-$Y$

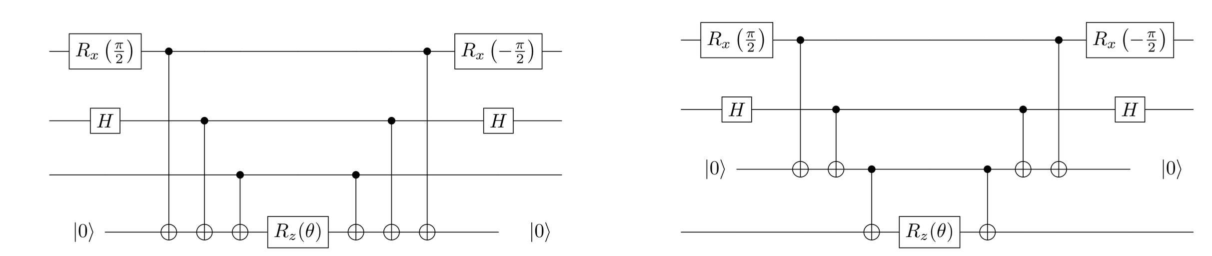



Figure 4. Two noise-resilient circuit that implements $\overline{R_{Y_1}}(\theta) = \exp\left(-i\theta Y_i X_{n-1} Z_n/2\right)$.

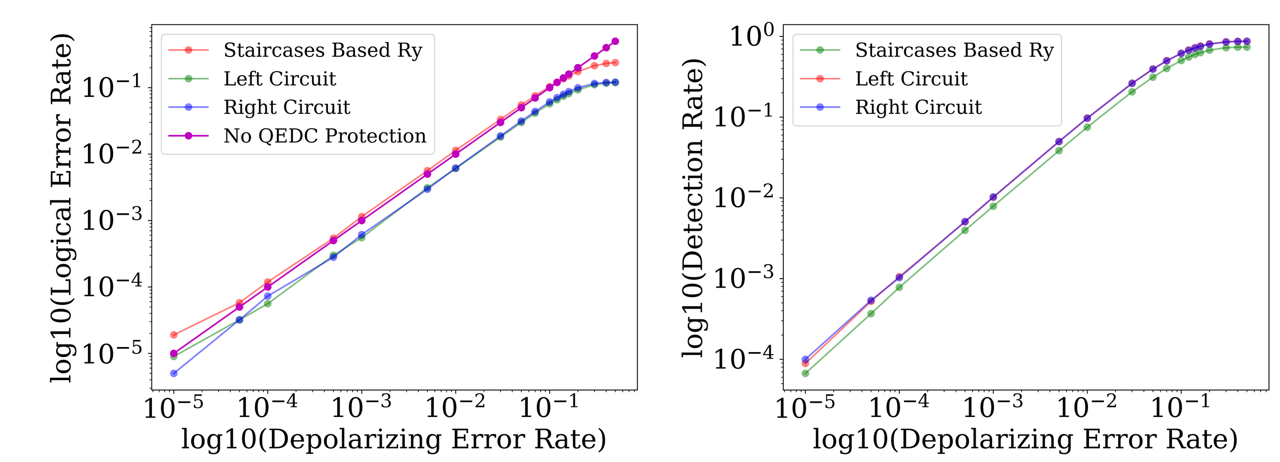



Figure 5. Logical error rate (left) and post-selection rate (right) of two noise-resilient circuits under 1,000,000 trials simulation with depolarizing gate noise.

### Logical Exponential with Weight $n > 3$ Pauli

Consider noise-resilient circuits for $\exp(-i\theta Z^{\otimes n})$ with different weights $n$.

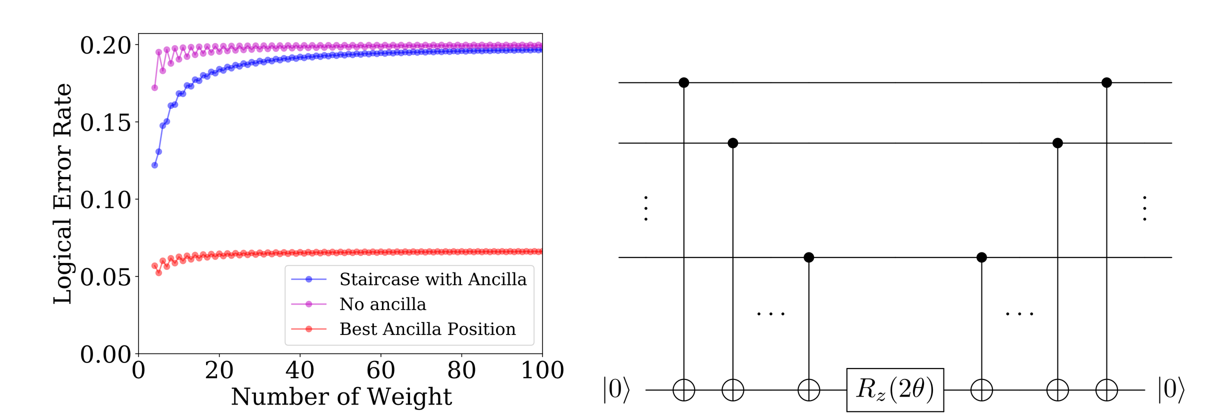



Figure 6. Logical error rate versus weights in $\exp(-i\theta Z^{\otimes n})$ using circuit on the right.

### Performance in Deep Circuit

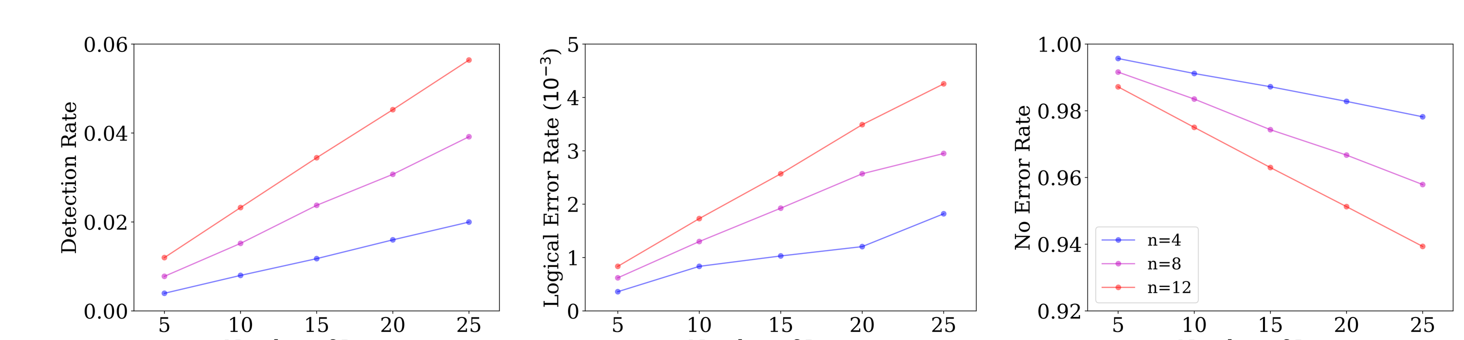



Figure 7. Detection rate, logical error rate and no error rate with different number of $\exp(-i\theta Z^{\otimes n})$ layers and qubit $n$ under error probability $10^{-4}$.

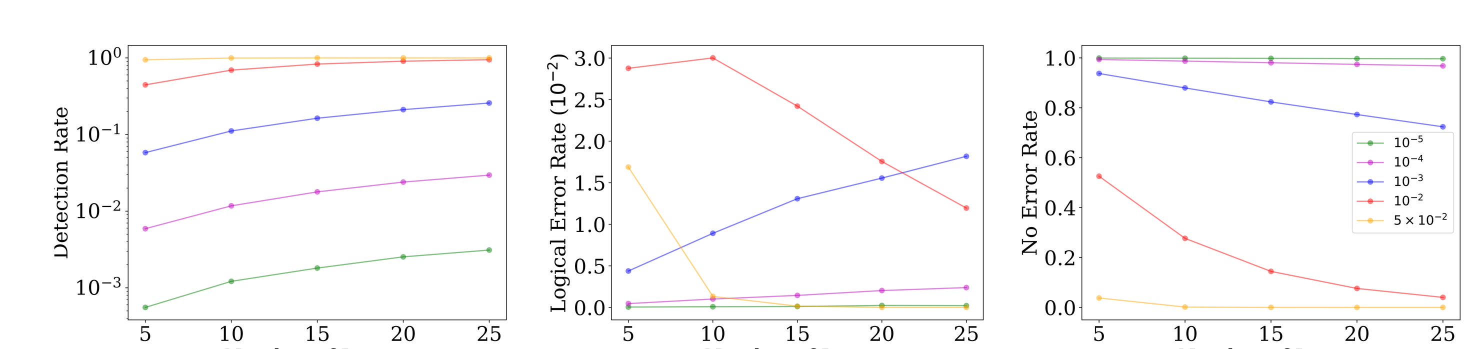



Figure 8. Detection rate, logical error rate and no error rate with different number of $\exp(-i\theta Z^{\otimes 6})$ layers and error probability.

Contact Information: daweiz@usc.edu for Dawei Zhong and tbrun@usc.edu for Todd Brun.