Introduction to Error Mitigation

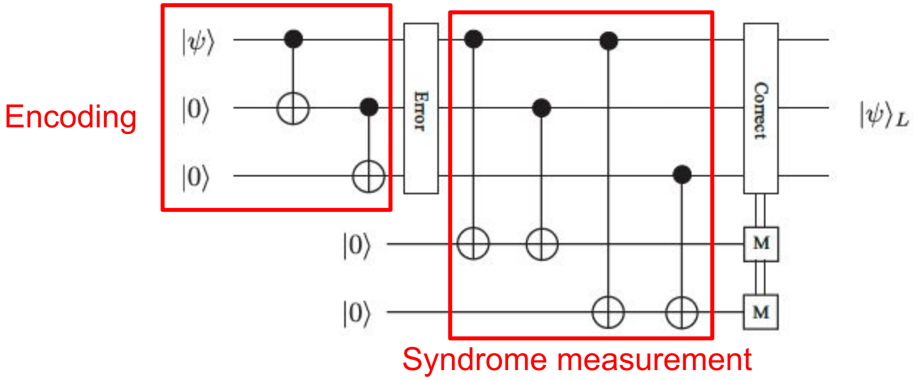
PHYS-513 Guest Lecture
Dawei Zhong, 02/23/2023

Content

- Quantum Error Correction and its limitation
- Zero Noise Extrapolation
- Probabilistic Error Cancellation
- Digital Dynamical Decoupling
- Readout Error Mitigation

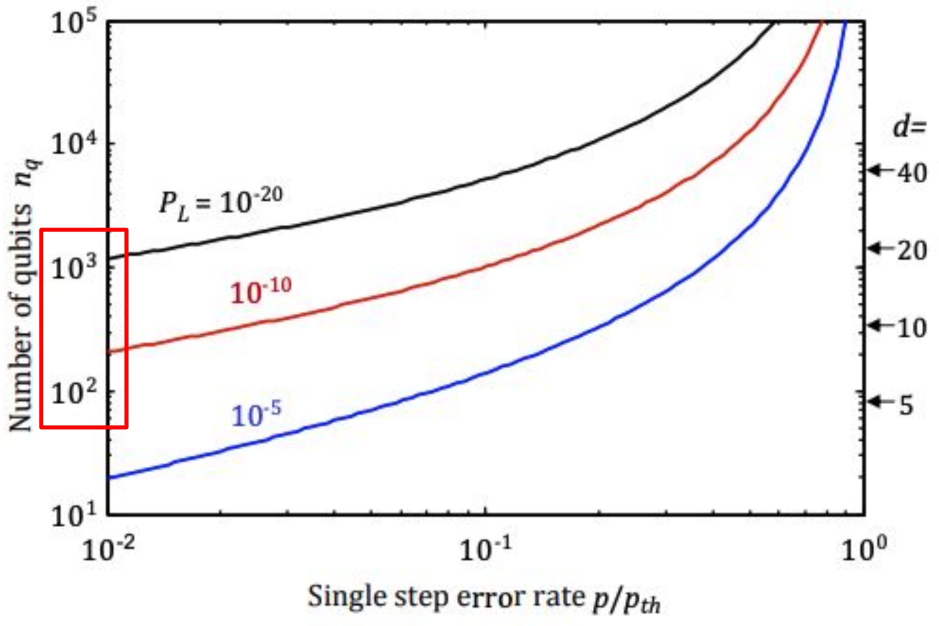
Error Correction and its limitation

- **NISQ era**: Current quantum computing processor are noisy
- Theoretically, we can correct noise by Quantum Error Correction Code (QECC)
  - Example: Encode 3 physical qubit into 1 logical qubits



Limitation of QEC on NISQ Device

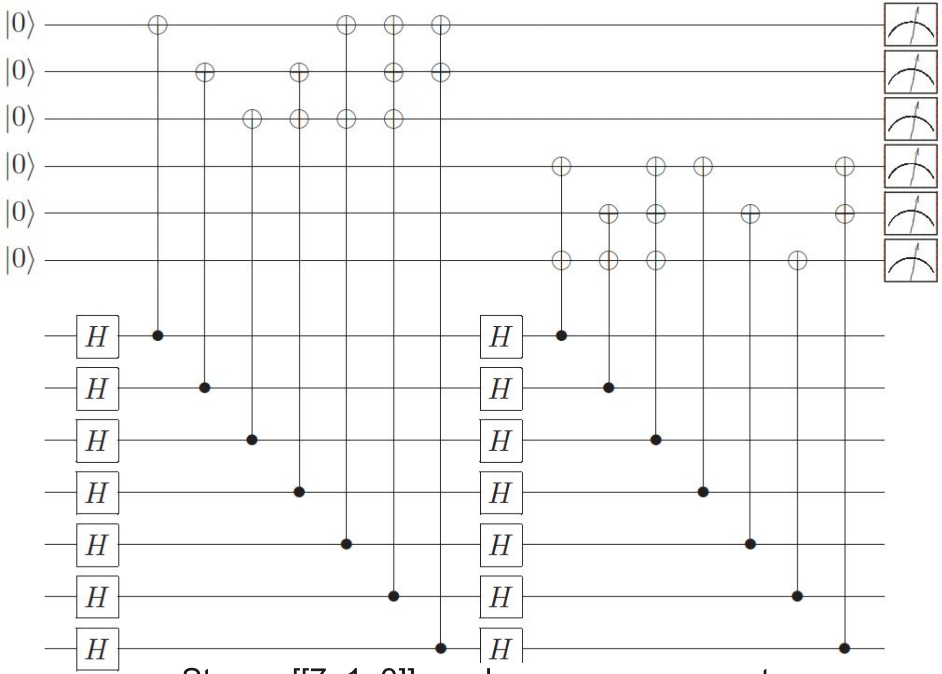
- **<u>Available number of qubits</u>**
- Weak connection
- Long logical operation



Fowler et al., Surface codes: Towards practical large-scale quantum computation, arXiv 1208.0928

Limitation of QEC on NISQ Device

- Available number of qubits
- <u>**Weak connection**</u>
- Long logical operation

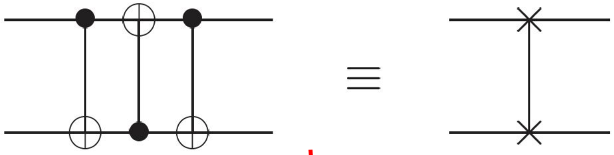


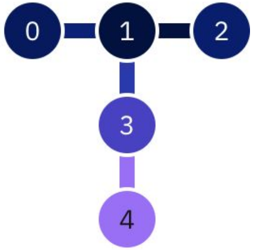
Steane [[7, 1, 3]] syndrome measurement

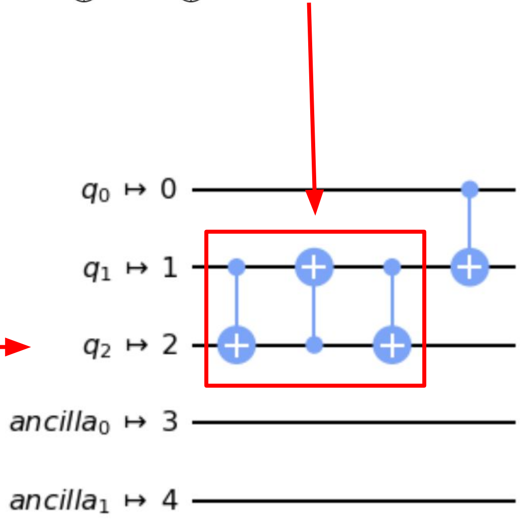
Nielsen and Chuang, Quantum Computation and Quantum Information, 2010

Limitation of QEC on NISQ Device

- Available number of qubits
- **Weak connection**
- Long logical operation

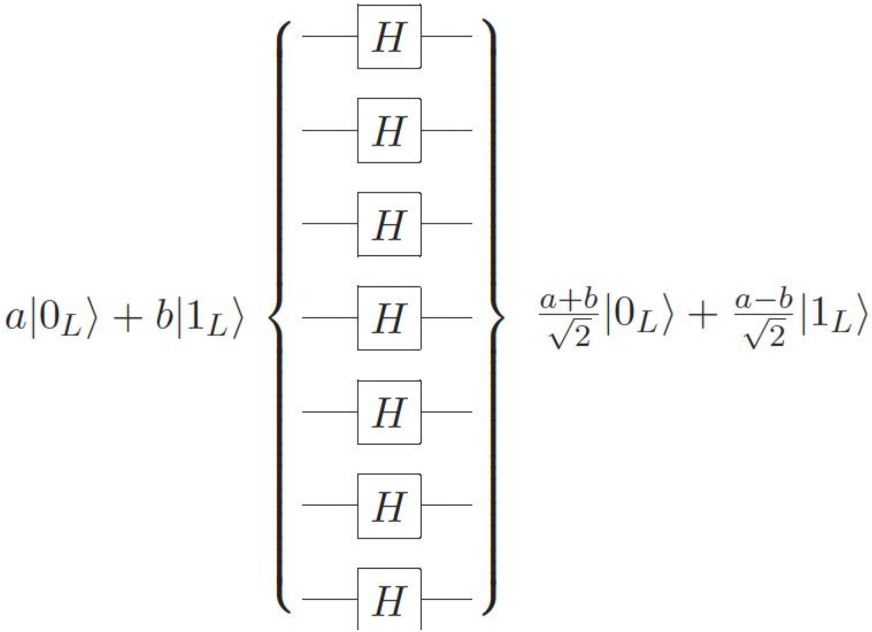






Limitation of QEC on NISQ Device

- Available number of qubits
- Weak connection
- <u>Long logical operation</u>

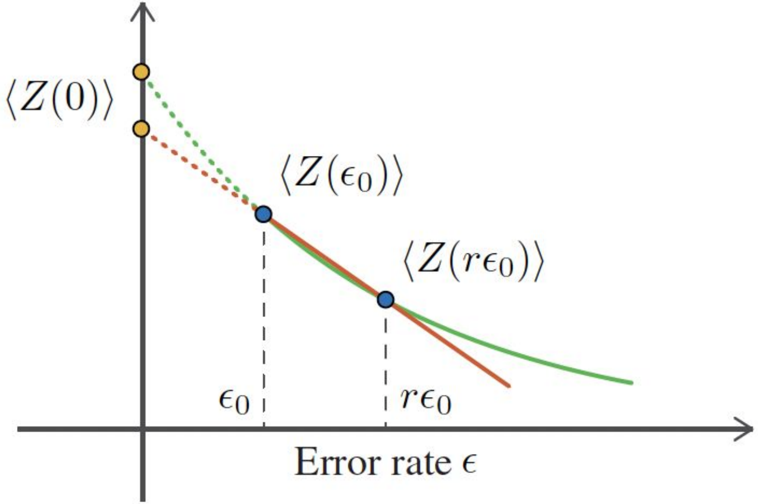


Logical Hadamard gate in Steane code

Nielsen and Chuang, Quantum Computation and Quantum Information, 2010

Zero Noise Extrapolation

- **Observable:** Expectation value $\langle\psi|\hat{O}|\psi\rangle$
- **Idea and first glance:** Repeat (part of) circuit to rescale the noise, extrapolate the noiseless expectation value by rescaling noise in quantum circuit

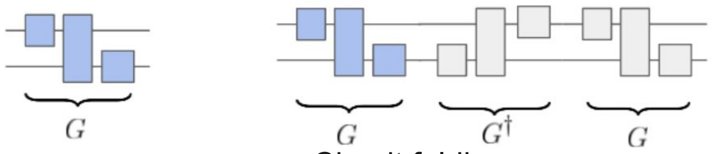


Ying Li and Simon Benjamin, Efficient Variational Quantum Simulator Incorporating Active Error Minimization, arXiv 1611.09301

Kristan Temme et al, Error mitigation for short-depth quantum circuits, arXiv 1612.02058

Suguru Endo et al, Practical Quantum Error Mitigation for Near-Future Applications, arXiv 1712.09271

Zero Noise Extrapolation



Circuit folding

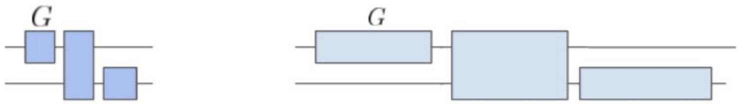
- **Construction:**
  - **Step 1:** Noise-scaling
    - Unitary folding (Circuit folding, gate folding)

Scale circuit from depth d to λd where $\lambda > 1$

$$U \to U(U^\dagger U)^n L_d L_{d-1} \cdots L_{d-s+1} L^\dagger_{d-s+1} \cdots L^\dagger_{d-1} L^\dagger_d$$

$$\lambda d = (2n+1)d + 2s$$

    - Parameter Noise Scaling
    - Pulse-stretching



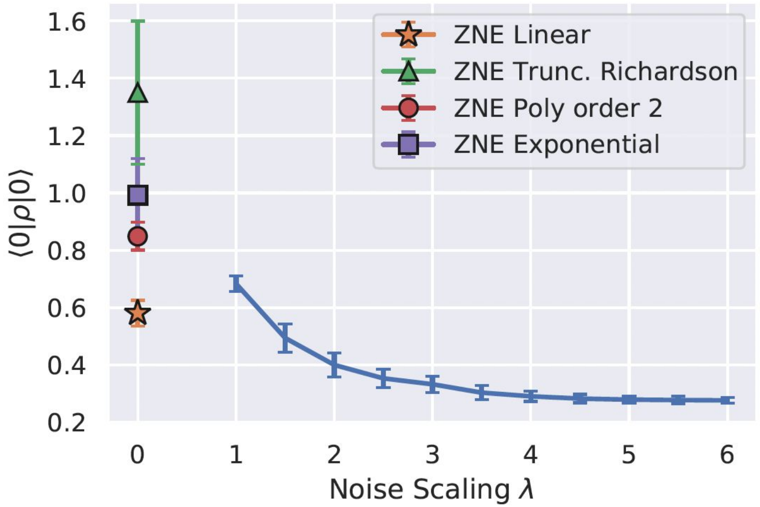
Pulse-stretching

Mitiq website

T Giurgica-Tiron et al, Digital zero noise extrapolation for quantum error mitigation, arXiv 2005.10921

Zero Noise Extrapolation

- **Construction:**
  - **Step 1:** Noise-scaling
  - **Step 2:** Extrapolation. Calculate expectation value for different scaling circuit, perform an extrapolation fit, obtain y-intercept



Zero Noise Extrapolation

Extrapolation

- Extrapolation with linear fit,

$$y = a + b\lambda$$

- Extrapolation with polynomial fit using `np.polyval`, it is required that $d + 1 \leq m$

$$y = c_0 + c_1\lambda + c_2\lambda^2 + \ldots + c_d\lambda^d$$

- Extrapolation with Richardson extrapolation. It is a particular case of a polynomial fit with order equal to the number of data points m minus 1,

$$y = c_0 + c_1\lambda + c_2\lambda^2 + \ldots + c_{m-1}\lambda^{m-1}$$

- Extrapolation with exponential fit,

$$y = a + b\exp(-c\lambda), c > 0$$

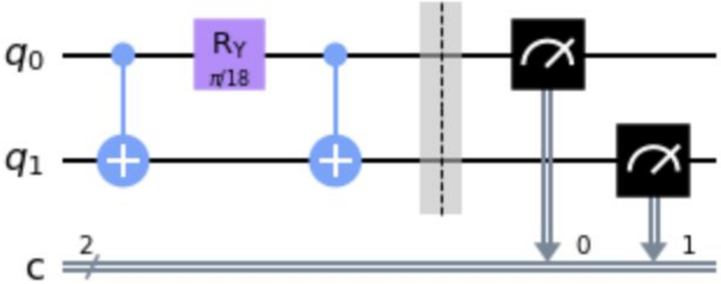
- Extrapolation with poly-exponential fit,

$$y = a \pm \exp[z(\lambda)], z(\lambda) = z_0 + z_1\lambda + \ldots + z_d\lambda^d$$

- Extrapolation with adaptive exponential, to reduce computational tasks

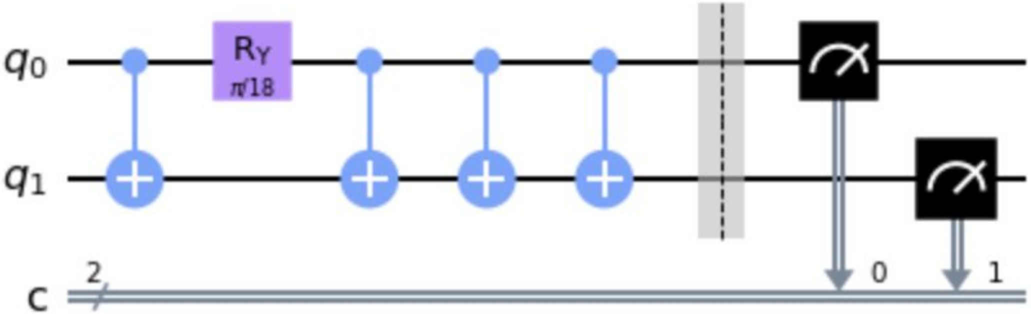
Zero Noise Extrapolation

- **Example:** $U \rightarrow U(U^{\dagger}U)^{n} L_d L_{d-1} \cdots L_{d-s+1} L^{\dagger}_{d-s+1} \cdots L^{\dagger}_{d-1} L^{\dagger}_{d}$



Original circuit (VQE)

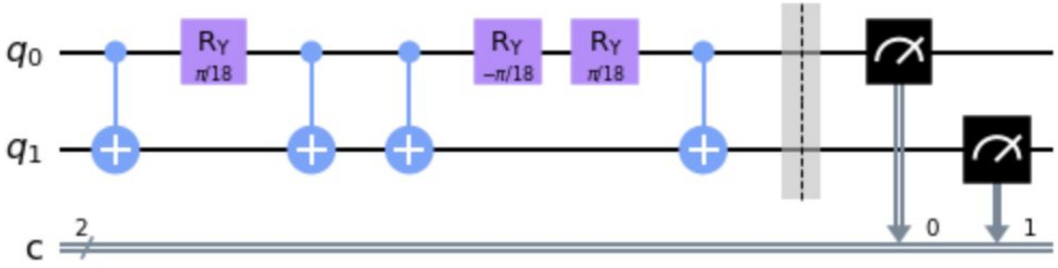
$$\lambda = 1, n = 0, s = 0$$



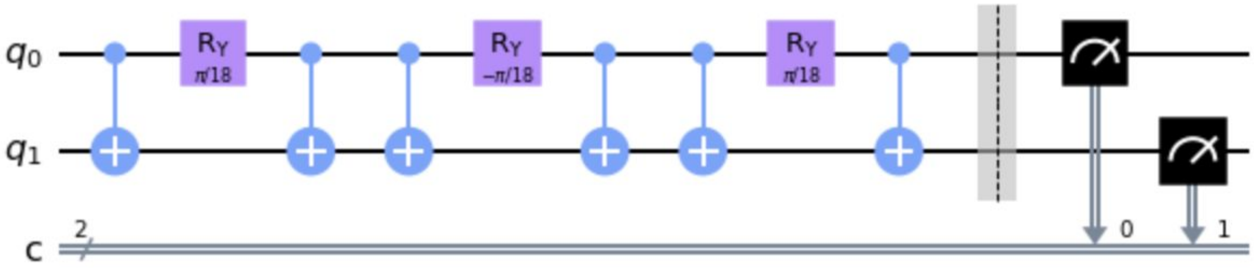
$$\lambda = 5/3, n = 0, s = 1$$

Zero Noise Extrapolation

- **Example:** $U \rightarrow U(U^{\dagger}U)^{n} L_d L_{d-1} \cdots L_{d-s+1} L_{d-s+1}^{\dagger} \cdots L_{d-1}^{\dagger} L_d^{\dagger}$



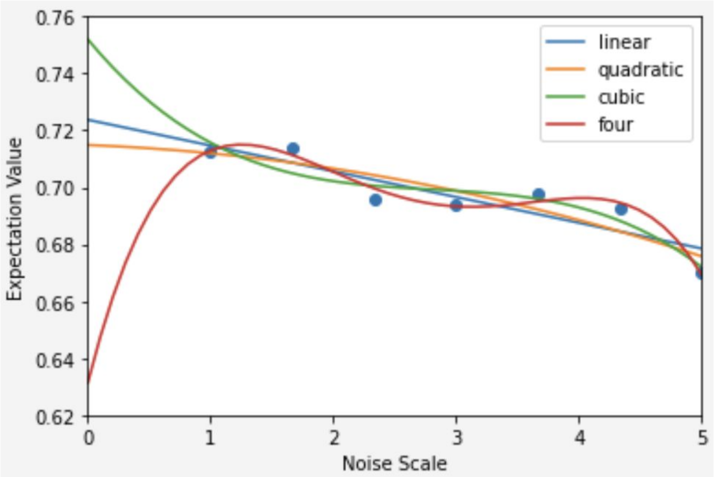
$$\lambda = 7/3, n = 0, s = 2$$



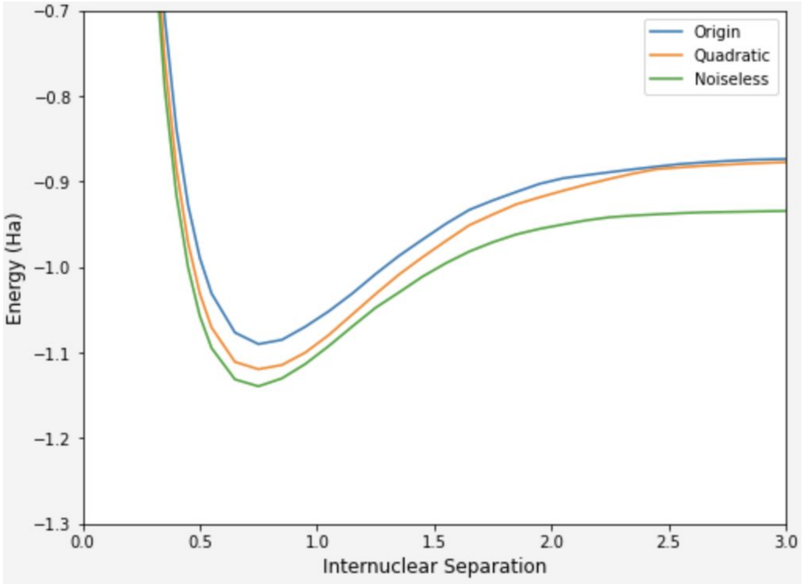
$$\lambda = 3, n = 1, s = 0$$

Zero Noise Extrapolation

- **Extrapolation:** Try polynomial extrapolation



- **Performance on H2 Dissociative Curve**



Probabilistic error cancellation

- **Theory:** Consider a circuit $\mathcal{U} = \mathcal{G}_t \circ \cdots \circ \mathcal{G}_2 \circ \mathcal{G}_1$
- **Linear combination:** Each gate can be expanded into a combination of implementable noisy operation

$$\mathcal{G}_i = \sum_{\alpha} \eta_{i,\alpha} \mathcal{O}_{i,\alpha}, \quad \eta_{i,\alpha} \in \mathbb{R}$$

  - Implementable operation set is large enough to approximately or exactly represent arbitrary unitary
  - Coefficients are allowed to take negative values.

Kristan Temme et al, Error mitigation for short-depth quantum circuits, arXiv 1612.02058

Probabilistic error cancellation

- Theory
- **Linear combination**
    - Single qubit implementable basis

TABLE I. Sixteen basis operations. Gates $[R_x]$ and $[R_y]$ can be derived from $[H]$ and $[S]$, and other operations can be derived from $[\pi]$, $[R_x]$, and $[R_y]$.

1	$[\mathbb{1}]$ (no operation)
2	$[\sigma^x] = [R_x]^2$
3	$[\sigma^y] = [R_x]^2[R_z]^2$
4	$[\sigma^z] = [R_z]^2$
5	$[R_x] = [(1/\sqrt{2})(\mathbb{1} + i\sigma^x)] = [H][S]^3[H]$
6	$[R_y] = [(1/\sqrt{2})(\mathbb{1} + i\sigma^y)] = [R_z]^3[R_x][R_z]$
7	$[R_z] = [(1/\sqrt{2})(\mathbb{1} + i\sigma^z)] = [S]^3$
8	$[R_{yz}] = [(1/\sqrt{2})(\sigma^y + \sigma^z)] = [R_x][R_z]^2$
9	$[R_{zx}] = [(1/\sqrt{2})(\sigma^z + \sigma^x)] = [R_z][R_x][R_z]$
10	$[R_{xy}] = [(1/\sqrt{2})(\sigma^x + \sigma^y)] = [R_x]^2[R_z]$
11	$[\pi_x] = [\frac{1}{2}(\mathbb{1} + \sigma^x)] = [R_z]^3[R_x]^3[\pi][R_x][R_z]$
12	$[\pi_y] = [\frac{1}{2}(\mathbb{1} + \sigma^y)] = [R_x][\pi][R_x]^3$
13	$[\pi_z] = [\frac{1}{2}(\mathbb{1} + \sigma^z)] = [\pi]$
14	$[\pi_{yz}] = [\frac{1}{2}(\sigma^y + i\sigma^z)] = [R_z]^3[R_x]^3[\pi][R_x]^3[R_z]$
15	$[\pi_{zx}] = [\frac{1}{2}(\sigma^z + i\sigma^x)] = [R_x][\pi][R_x]^3[R_z]^2$
16	$[\pi_{xy}] = [\frac{1}{2}(\sigma^x + i\sigma^y)] = [\pi][R_x]^2$

Suguru Endo et al, Practical Quantum Error Mitigation for Near-Future Applications, arXiv 1712.09271

Probabilistic error cancellation

- **Linear combination**

$$\mathcal{G}_i = \sum_{\alpha} \eta_{i,\alpha} \mathcal{O}_{i,\alpha}, \quad \eta_{i,\alpha} \in \mathbb{R}$$

- **Quasi-probability (MCMC)**

$$\sum_{\alpha} \eta_{i,\alpha} = 1, \qquad \gamma_i = \sum_{\alpha} |\eta_{i,\alpha}| \geq 1$$

- **Error Cancellation**

$$\langle A \rangle_{\text{ideal}} = \text{tr}[A\mathcal{U}(\rho_0)] = \sum_{\vec{\alpha}} \eta_{\vec{\alpha}} \langle A_{\vec{\alpha}} \rangle_{\text{noisy}}$$

Kristan Temme et al, Error mitigation for short-depth quantum circuits, arXiv 1612.02058

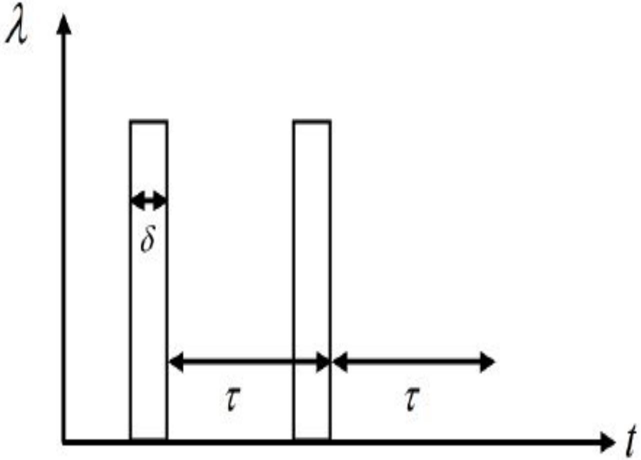
Digital Dynamical Decoupling

- **Functionality:** Cancel low frequency, decoherence noise
- **Simple illustration:** In physical level,

Hamiltonian of system and its interaction with surrounding environment

$$H_S = \lambda(t)X,\ H_{SB} = Z \otimes B_z$$

- When no pulse, free evolution for H_{SB}, evolution operator $f_\tau = \exp(-iH_{SB}\tau)$
- When with pulse at time interval $[t, t+\delta]$, evolution operator $X = \exp(-i\delta\lambda X \otimes I_B)$
- The evolution operator $Xf_\tau X^\dagger f_\tau = I$

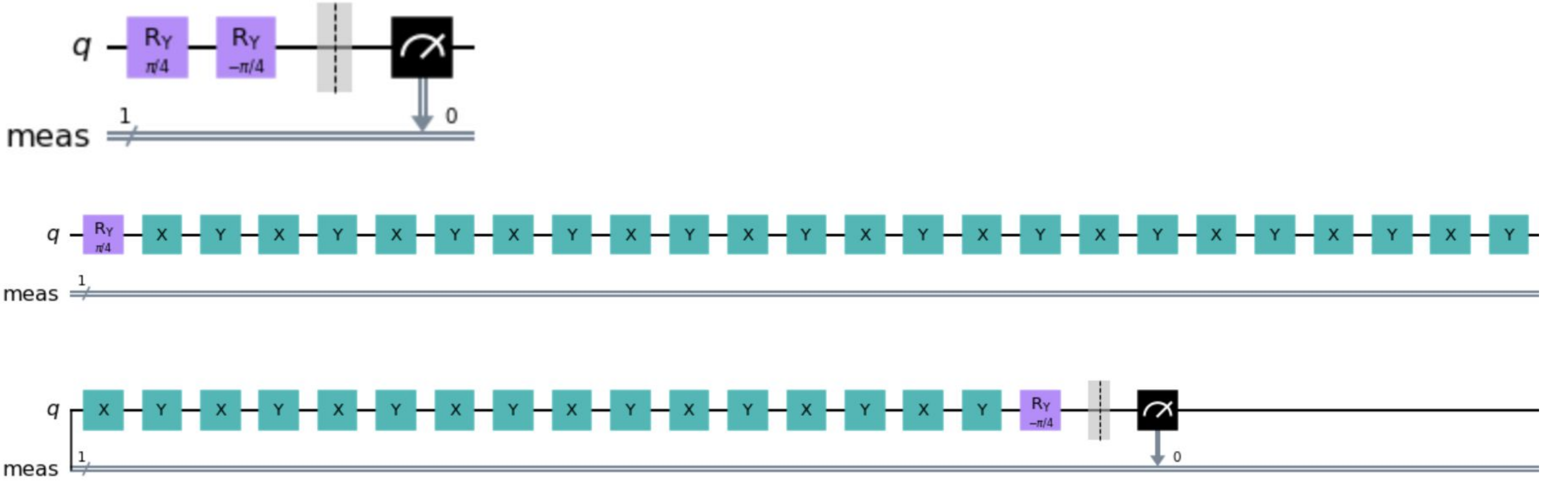


Illustration

Daniel Lidar, Quantum Error Correction Lecture Note, 2021

Digital Dynamical Decoupling

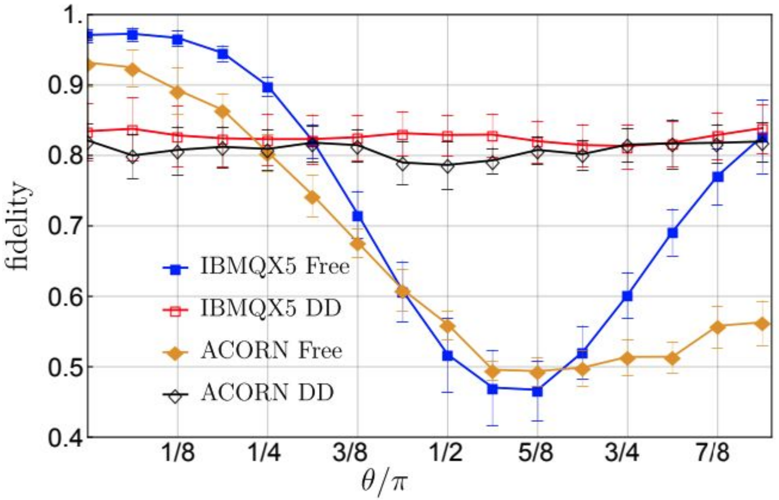
- **Implementation:** XY4 Sequence

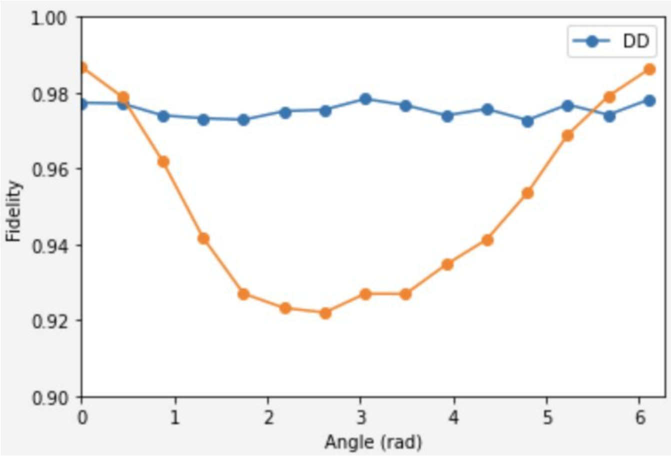


Bibek Pokharel et al, Demonstration of fidelity improvement using dynamical decoupling with superconducting qubits, arXiv 1807.08768

Digital Dynamical Decoupling

- **Result:** XY4 Sequence

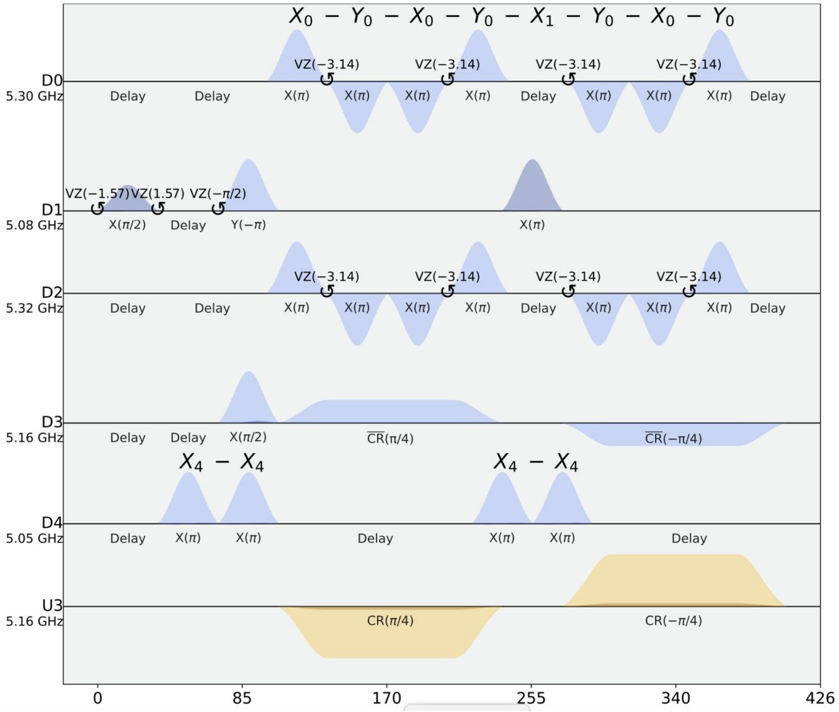


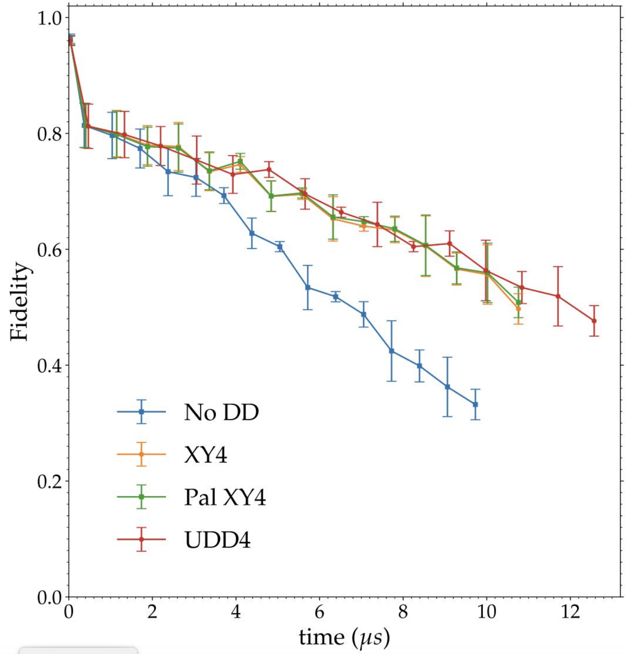


Bibek Pokharel et al, Demonstration of fidelity improvement using dynamical decoupling with superconducting qubits, arXiv 1807.08768

Digital Dynamical Decoupling

- **More Application:**

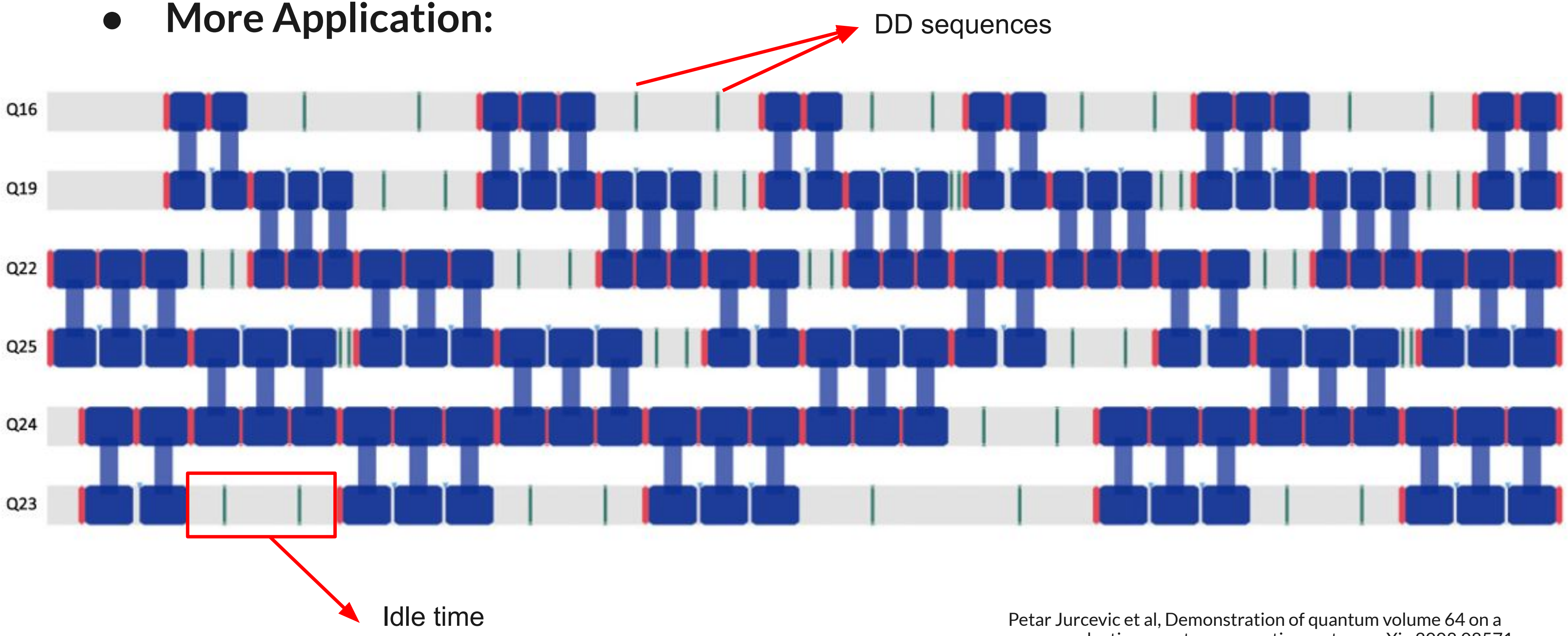




Vinay Tripathi et al, Suppression of crosstalk in superconducting qubits using dynamical decoupling, arXiv 2108.04530

Digital Dynamical Decoupling

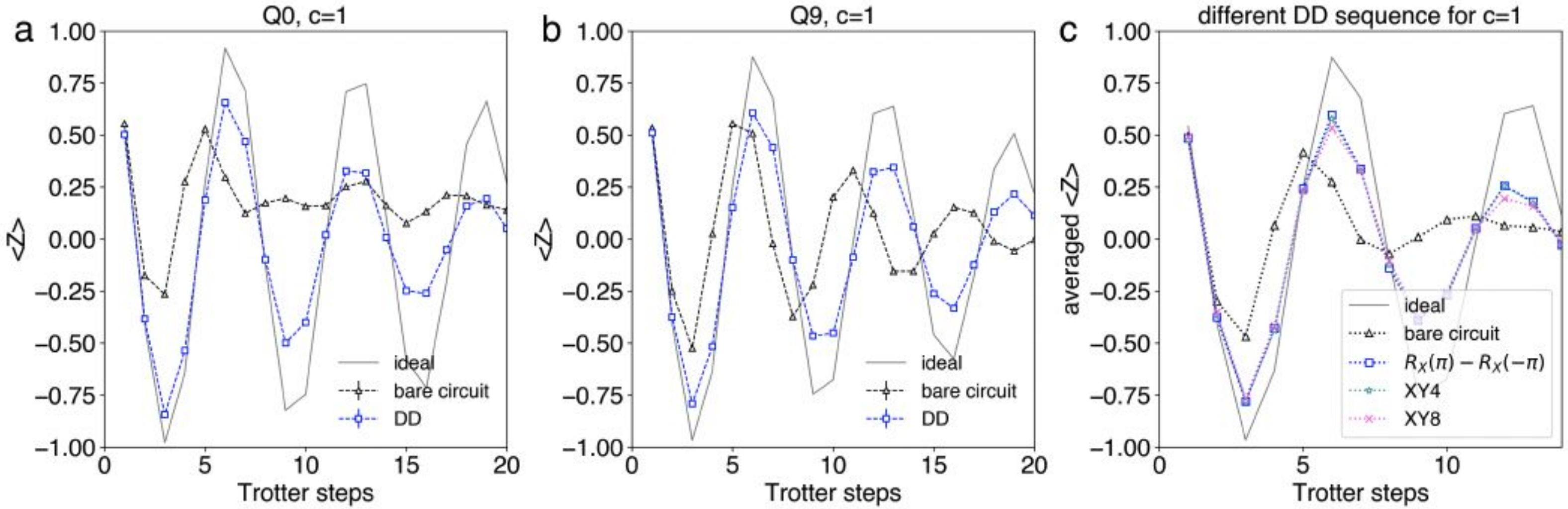
- More Application:



Petar Jurcevic et al, Demonstration of quantum volume 64 on a superconducting quantum computing system, arXiv 2008.08571

Digital Dynamical Decoupling

- **More Application:** Also use with other error reduction techniques



Youngseok Kim et al, Scalable error mitigation for noisy quantum circuits produces competitive expectation values, arXiv 2108.09197

Measurement Error Mitigation

- Measurement PDF and true PDF (2 Qubits case as example):

$$p_{\text{meas}} = (\Pr[\text{Meas } 00] \quad \Pr[\text{Meas } 01] \quad \Pr[\text{Meas } 10] \quad \Pr[\text{Meas } 11])^T$$

$$p_{\text{real}} = (\Pr[\text{True } 00] \quad \Pr[\text{True } 01] \quad \Pr[\text{True } 10] \quad \Pr[\text{True } 11])^T$$

- Response Matrix

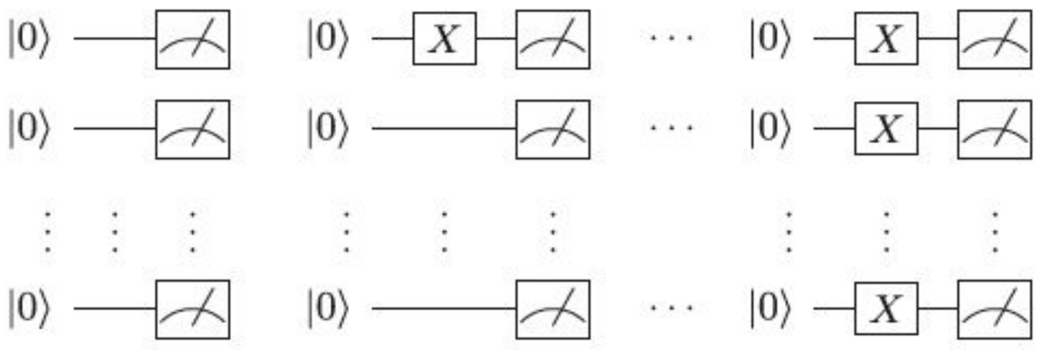
$$p_{\text{meas}} = A p_{\text{real}}$$

Measurement Error Mitigation

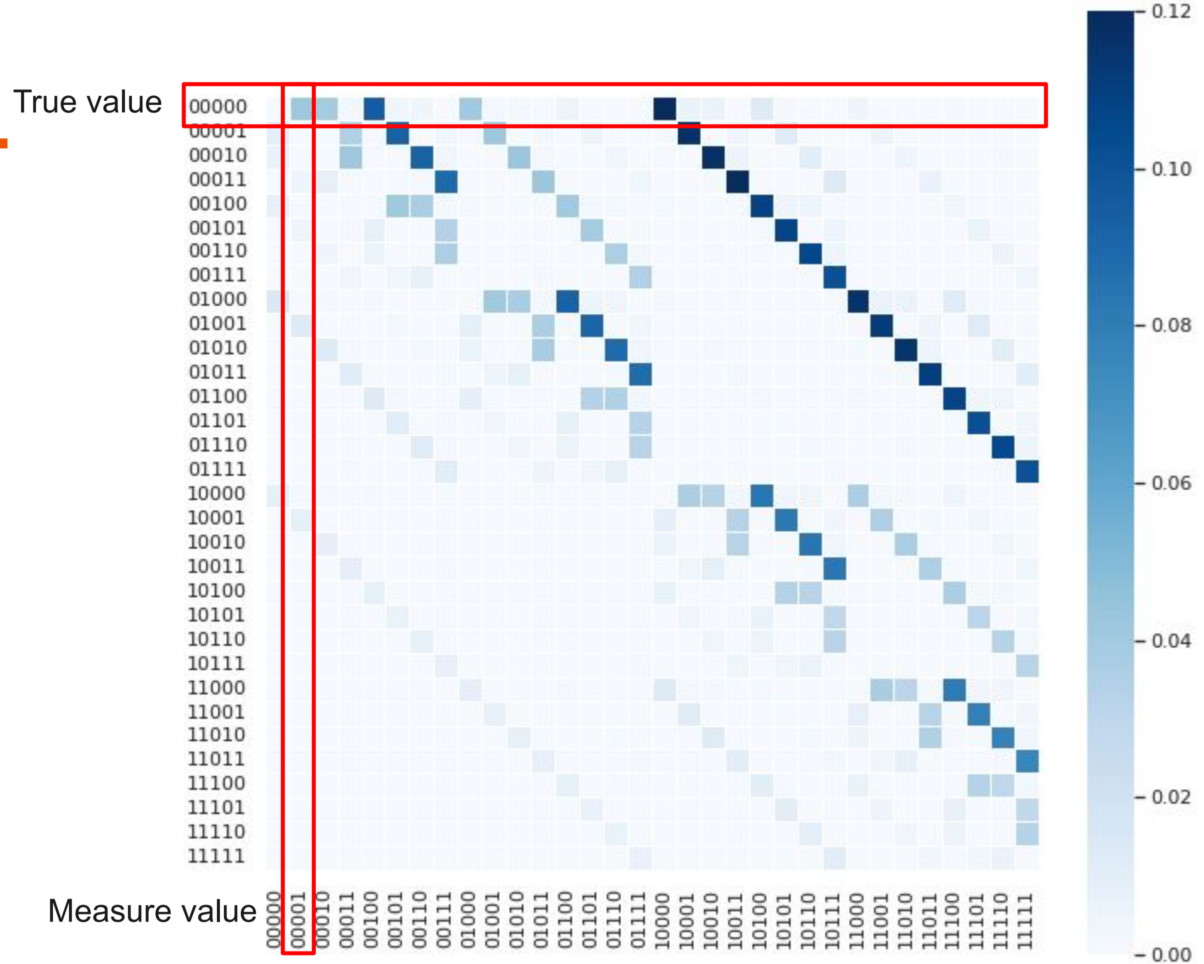
- Response Matrix (2 Qubits case as example)

$$A = \begin{pmatrix} \Pr[\text{Meas } 00|\text{True } 00] & \Pr[\text{Meas } 01|\text{True } 00] & \Pr[\text{Meas } 10|\text{True } 00] & \Pr[\text{Meas } 11|\text{True } 00] \\ \Pr[\text{Meas } 00|\text{True } 01] & \Pr[\text{Meas } 01|\text{True } 01] & \Pr[\text{Meas } 10|\text{True } 01] & \Pr[\text{Meas } 11|\text{True } 01] \\ \Pr[\text{Meas } 00|\text{True } 10] & \Pr[\text{Meas } 01|\text{True } 10] & \Pr[\text{Meas } 10|\text{True } 10] & \Pr[\text{Meas } 11|\text{True } 10] \\ \Pr[\text{Meas } 00|\text{True } 11] & \Pr[\text{Meas } 01|\text{True } 11] & \Pr[\text{Meas } 10|\text{True } 11] & \Pr[\text{Meas } 11|\text{True } 11] \end{pmatrix}$$

- Response Matrix preparation



Response Matrix (5 Qubits case)



Measurement Error Mitigation

- Tensor Error Model

$$A_0 = \begin{pmatrix} \Pr(\text{measure } 0|\text{true } 0) & \Pr(\text{measure } 0|\text{true } 1) \\ \Pr(\text{measure } 1|\text{true } 0) & \Pr(\text{measure } 1|\text{true } 1) \end{pmatrix} = \begin{pmatrix} 1-p & q \\ p & 1-q \end{pmatrix}$$

$$A = A_0 \otimes A_1 \otimes \cdots \otimes A_n = \bigotimes_{i=0}^{n} A_i = \bigotimes_{i=0}^{n} A_i \begin{pmatrix} 1-p_i & q_i \\ p_i & 1-q_i \end{pmatrix}$$

- Continuous time Markov processes
- Others

Data from IBM lima processor

Qubit	Prob meas0 prep1	Prob meas1 prep0
0	0.029	0.0084
1	0.0268	0.005
2	0.0302	0.0104
3	0.0502	0.017
4	0.0832	0.0126

Sergey Bravyi et al, Mitigating measurement errors in multi-qubit experiments, arXiv 2006.14044

Measurement Error Mitigation

- **Correct the error:**
  - Inverse matrix

$$A_{ij} = p(\text{measure } i | \text{true } j) \iff A^{-1}_{ij} = p(\text{true } j | \text{measure } i)$$

  - Least Square

$$\mathbf{p}_{\text{exp}} = A\mathbf{p}_{\text{ideal}} \iff \mathbf{p}_{\text{ideal}} = \arg \min_{||\mathbf{p}_{\text{ideal}}||} ||\mathbf{p}_{\text{exp}} - A\mathbf{p}_{\text{ideal}}||^2$$

  - Iterative bayes unfolding

Benjamin Nachman et al, Unfolding Quantum Computer Readout Noise,
arXiv 1910.01969

Measurement Error Mitigation

- **Correct the error:**
  - Inverse matrix
  - Least Square
  - Iterative bayes unfolding

$$\Pr^{(n+1)}_{\text{real}}(\text{True is } i) = \sum_j \Pr(\text{true } i|\text{measure } j) \times \Pr(\text{measure } j)$$

$$= \sum_j \frac{A_{ji}\Pr^{(n)}_{\text{real}}(\text{True is } i)}{\sum_k A_{jk}\Pr^{(n)}_{\text{real}}(\text{True is } k)} \times \Pr(\text{measure } j)$$

Benjamin Nachman et al, Unfolding Quantum Computer Readout Noise,
arXiv 1910.01969